JOURNAL OF AIRCRAFT
Vol. 45, No. 4, July–August 2008

Design Forum

DESIGN FORUM papers range from design case studies to presentations of new methodologies to speculations about emerging design trends. They vary from 2500 to 12,000 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Simplified Thrust and Fuel Consumption Models for Modern Two-Shaft Turbofan Engines

Matthias Bartel*
Rolls–Royce Deutschland, 15827 Dahlewitz, Germany
and
Trevor M. Young†
University of Limerick, Limerick, Ireland

DOI: 10.2514/1.35589

Performance data of two-shaft turbofan engines have been used to investigate the validity of previously published models (empirical equations) that describe an engine's thrust variation as a function of Mach number and altitude during takeoff and climb. Where necessary, the constants and the format of the models have been revised. An updated model is presented that describes the takeoff thrust to within ±1% of that of the reference engines, for flight speeds up to Mach 0.4. This takeoff thrust model has been adapted to account for the impact of bleed air extraction and altitude effects. To describe the maximum climb thrust, a new approach has been devised in which the climb path has been divided into three segments and an equation developed for each segment, based on a reference thrust at 30,000 ft and typical climb/speed schedules. A widely used thrust specific fuel consumption power law model for cruise has been investigated, and new empirical constants determined. These results will allow a more realistic prediction of engine performance for the purpose of preliminary aircraft design or initial performance analysis.

Nomenclature

C_{19} = velocity of cold gas stream, m/s
C_9 = velocity of hot gas stream, m/s
c_p = specific heat capacity of engine air at constant pressure, J/kg · K
c_1 = empirical constant
D_{fan} = fan diameter, in.
F = net thrust, lbf
F_0 = net thrust at sea-level static, lbf
F_{30} = net thrust at 30,000 ft, Mach 0.79 (V_{cas} 300 kt), lbf
F^* = net thrust at optimum L/D, lbf
G = gas generator function
G_0 = gas generator function at sea-level static
G_{30} = gas generator function at 30,000 ft, Mach 0.79
h = specific enthalpy, kJ/kg
K_1, K_2 = empirical constants introduced in Eq. (7)
L/D = lift to drag ratio
M = flight Mach number
$\dot{m}$ = mass flow, kg/s
P = power, W
p_{amb} = ambient pressure, kPa
p_{amb0} = ambient pressure at sea level, kPa
p_{amb30} = ambient pressure at 30,000 ft, kPa
s = specific entropy, kJ/kg K
s = thrust specific fuel consumption, lb/hr/lbf
s_1 = reference cruise thrust specific fuel consumption, lb/hr/lbf
s^* = reference cruise thrust specific fuel consumption at optimum L/D, lb/hr/lbf
T_0 = ambient air temperature, K
V_{cas} = calibrated air speed, kt
V_{casref} = 300 kt calibrated air speed, kt
W_1 = engine inlet mass flow, lb/s
W_1^* = engine inlet mass flow at optimum L/D, lb/s
λ = bypass ratio
θ = temperature ratio T/T_0

Subscripts

hot = core
cold = bypass air

Presented as Paper 7847 at the 7th AIAA ATIO Conference, 2nd CEIAT International Conference on Innovation & Integration in Aerospace Sciences, 17th LTA Systems Technology Conference; followed by 2nd TEOS Forum, Belfast, Northern Ireland, 18–20 September 2007; received 9 November 2007; revision received 27 February 2008; accepted for publication 27 February 2008.

*Development Engineer, Performance Department. Member of AIAA.
†Senior Lecturer in Aeronautical Engineering, Department of Mechanical and Aeronautical Engineering. Senior Member of AIAA.

I. Introduction

A NUMBER of models (or empirical equations or laws, as they are sometimes called) exist that describe the variation of the thrust of a turbofan engine with forward speed and altitude (as described by [1–8], for example). Similarly, expressions exist that describe the thrust specific fuel consumption (TSFC) of a turbofan engine in cruise ([2,3,7–10], for example). These relatively simple expressions usually take a polynomial or power law form, and include a few constants that have to be determined from actual engine data before the model can be used. Typically, such expressions provide a means for engineers (without access to sophisticated engine performance databases or software) to obtain an estimate of the engine's thrust or TSFC for the purpose of conceptual airplane design or preliminary performance analysis.

In this study, a few selected models (empirical equations) were assessed using actual engine performance data, in which the empirical constants were determined using curve fitting techniques and, in certain cases, the form of equations has been revised to better suit the available engine data. Four two-shaft turbofan engines with different layouts (i.e., mixed, unmixed, boosted, unboosted) were used for this investigation, covering thrust ranges from 10,000 up to 25,000 lbf (and about 20 years of engine development). The performance data used in this study were taken from engine models of older, existing engines with bypass ratios of three and higher, as well as from current developments with higher bypass ratios of up to nine. The overall pressure ratio was in the range of 18–36.

II. Objectives

The overall objective of this investigation was to evaluate previously published models (empirical equations) that describe engine performance (specifically the engine *thrust* and *thrust specific fuel consumption*) using actual performance data for modern two-shaft turbofan engines. The investigated equations will be updated to better represent current applications. The work is split into three different parts, corresponding to the takeoff (see Sec. IV), climb (Sec. V), and cruise (Sec. VI) segments of a typical mission of a passenger jet transport airplane. The specific objectives are given as follows.

1) For maximum takeoff thrust representation, published models will be assessed, and the most promising model updated to best describe the thrust variation with Mach number, where the control parameter is constant turbine entry temperature (TET). The impact of customer bleed air extraction and altitude on the engine thrust will then be considered within the revised model.

2) For the climb, a new approach will be explored: the climb is divided into three segments (corresponding to typical climb/speed schedules) and, for each segment, an equation will be developed to describe the maximum climb thrust in which the reference thrust is taken as the thrust at 30,000 ft (and not sea level, as is usually done).

3) For the cruise, the thrust will not be considered (as it is typically equal to the airplane's drag); instead, a TSFC model (in which the influence of flight Mach number and altitude is given) will be investigated and, where possible, revised.

III. Usable Power of a Turbofan Engine

The thrust models developed for the takeoff (Sec. IV) depend on a variable designated as G, which is known as the *gas generator function*. This parameter G, which represents the usable power of a turbofan engine, has previously been used by Torenbeek [1] to describe the thrust produced by the engine. The gas generator function can be derived in different ways; the simplest definition can be determined directly from an $h - s$ (i.e., specific enthalpy vs specific entropy) diagram (as illustrated in Fig. 1), and this approach was adopted for this study. The usable power, which is the time rate that energy (or work) is "delivered," can be expressed as the sum of the usable powers within the hot and cold gas streams, i.e.,

$$P_{\text{usable}} = P_{\text{hot}} + P_{\text{cold}} \tag{1}$$

The available power depends on the mass flow of the gas streams through the nozzles and their corresponding speeds, thus

$$P_{\text{hot}} = \frac{1}{2}\dot{m}_{\text{hot}}C_9^2 \tag{2}$$

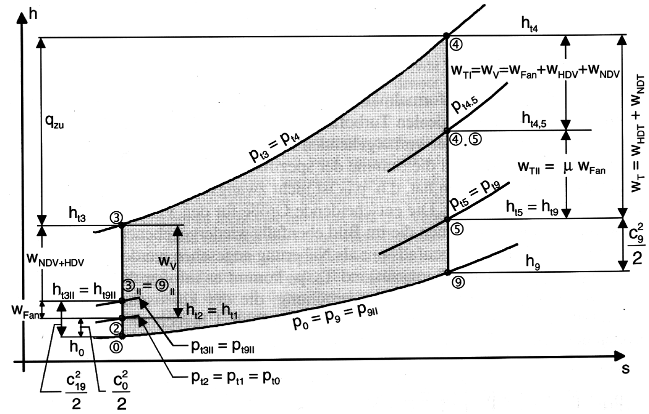

Fig. 1 Specific enthalpy *h* vs specific entropy *s* diagram for a two-shaft bypass engine (redrawn after [11]).

and

$$P_{\text{cold}} = \frac{1}{2}\dot{m}_{\text{cold}}C_{19}^2 \tag{3}$$

The hot and cold gas streams are linked by the bypass ratio λ, i.e.,

$$\dot{m}_{\text{cold}} \approx \lambda\dot{m}_{\text{hot}} \tag{4}$$

These expressions [Eqs. (2–4)] are now substituted into Eq. (1), and the usable power nondimensionalized in the identical manner to that undertaken by Torenbeek [1], to define the gas generator function.

$$G = \frac{(1/2)\dot{m}_{\text{hot}}(C_9^2 + \lambda C_{19}^2)}{\dot{m}_{\text{hot}}c_pT_0} \tag{5}$$

And this expression reduces to give the required result:

$$G = \frac{C_9^2 + \lambda C_{19}^2}{2c_pT_0} \tag{6}$$

The hot and cold stream velocities and the bypass ratio of an engine can be easily derived by performance calculation. Typical values for G are in the range of 0.8–1.2 for engines with bypass ratios greater than four (further details regarding G are given in Sec. IV).

IV. Takeoff Thrust

The influence on the takeoff thrust of three parameters has been investigated: forward speed (Mach number), bleed air extraction, and altitude.

A. Thrust vs Flight Mach Number at Sea Level

The available takeoff thrust of a particular engine depends on the philosophy used for creating the engine ratings. Two commonly used parameters for controlling the rating "shape" are TET (turbine entry temperature) and NL (low-pressure shaft speed). The results presented in this part of the paper are based on the assumption of a constant TET vs Mach number relationship.

The thrust F divided by the static thrust at sea level F_0, without considering the impact of bleed air extraction, follows the general behavior displayed in Fig. 2. This thrust vs Mach number relationship can be represented, with reasonable accuracy, by a quadratic function, as described in [1,8], for example

$$\frac{F}{F_0} = 1 - K_1M + K_2M^2 \tag{7}$$

where K_1 and K_2 are empirical, correlation factors selected to best fit the engine data.

To expand the applicability of this relationship, the numeral 1 in Eq. (7) has, herein, been replaced by a variable A (where $A \leq 1$) to take into account the effect of bleed air and altitude on the engine thrust; hence, Eq. (7) becomes

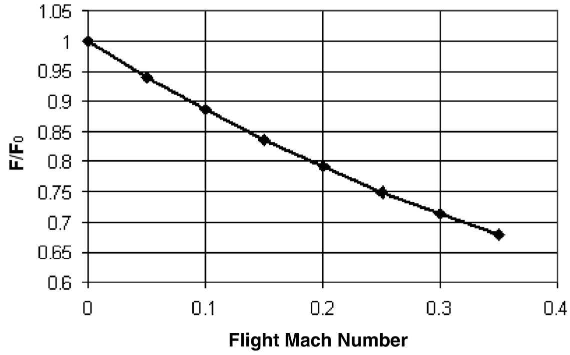

Fig. 2 Typical thrust vs Mach number at sea level.

$$\frac{F}{F_0} = A - K_1 M + K_2 M^2 \tag{8}$$

The parameter A is explored in Sec. IV.B.

The usefulness of Eqs. (7) and (8) can be enhanced if the factors K_1 and K_2 can be related to fundamental engine design parameters, and this was the approach adopted by Torenbeek [1], as he defined K_1 and K_2 in terms of G. However, his approach for calculating G is based on component efficiencies and some empirical constants, which, it was speculated by the authors, may not be particularly accurate for modern gas turbine engines. His equation for calculating the takeoff thrust is

$$\frac{F}{F_0} = 1 - \frac{0.454(1+\lambda)}{\sqrt{(1+0.75\lambda)G}} M + \left(0.6 + \frac{0.13\lambda}{G}\right) M^2 \tag{9}$$

When the accuracy of this expression was assessed against the available engine data, it was noted that it only provided a good accuracy for Mach numbers below about 0.2. The reason for this was thought to be the relatively poor estimation of the gas generator function G. It was thus decided to determine "best-fit" values of K_1 and K_2 for the available engine performance data, which comprised real (i.e., existing) and predicted (i.e., theoretical) engine data, by assuming a constant TET vs Mach number relationship. The results are given in Figs. 3 and 4. It is evident from these two figures that the investigated engines are located in two regions of specific thrust (i.e., F_0/W_1), which has been identified to be the best parameter for correlation. Using these results and the definition of G, as given by Eq. (6), Torenbeek's thrust equation [Eq. (9)] was modified to represent the performance of new aeroengines over a wider Mach number range. The resulting equation is

$$\frac{F}{F_0} = 1 - \frac{0.377(1+\lambda)}{\sqrt{(1+0.82\lambda)G_0}} M + (0.23 + 0.19\sqrt{\lambda}) M^2 \tag{10}$$

In Eq. (10), G_0 and λ are fixed reference values at sea-level static (SLS) conditions, which have to be known for the engine being modeled. A correlation between G_0 and bypass ratio is given in Fig. 5, which is expected to be roughly valid for two-shaft engines.

The accuracy of Eq. (10) was assessed by comparing the thrust values for all investigated engines to the thrust values obtained from Eq. (10). This produced a correlation that was better than $\pm 1\%$ for flight speeds up to Mach 0.4 (Fig. 6).

As can be seen in Fig. 7, the equation developed during this work fits the engine performance data very well across a wide range of flight Mach numbers, whereas Torenbeek's original approach [1] [i.e., Eq. (9)] only yields a good accuracy below about Mach 0.2. However, it should be noted that there is a small inconsistency in the comparison: the value of G used in Eq. (9) for this comparison was calculated using Eq. (6), and not using Torenbeek's original method for calculating G.

B. Effect of Aircraft System Bleed Air Extraction on Thrust

Air is usually taken from the high-pressure (HP) compressor on two-shaft engines to meet the aircraft's bleed air requirements (also

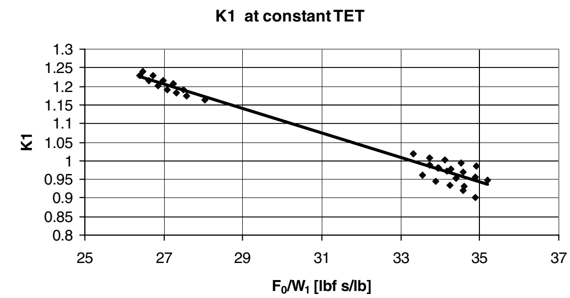

Fig. 3 K_1 for different aeroengines, dependent on specific thrust.

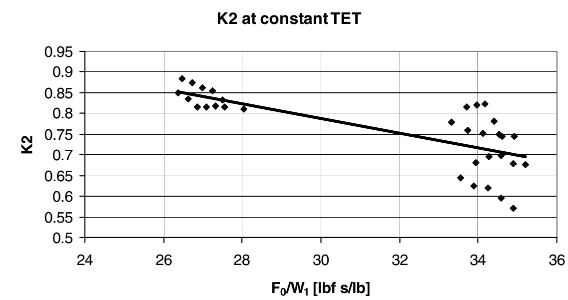

Fig. 4 K_2 for different aeroengines, dependent on specific thrust.

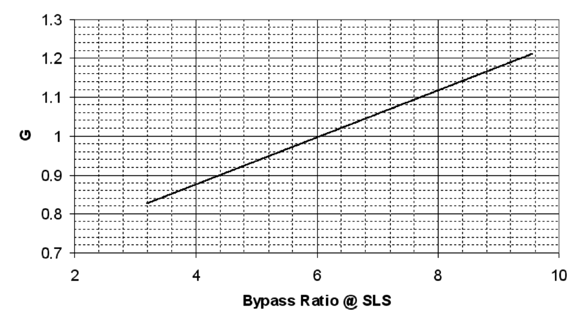

Fig. 5 Approximation for G vs bypass ratio for two-shaft engines from Eq. (6).

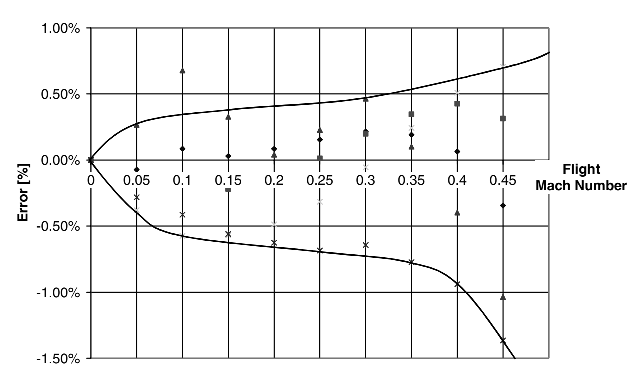

Fig. 6 Deviation of thrust calculated using Eq. (10) from that obtained from four different performance engine models, as a function of Mach number.

known as customer bleed air). The extraction of air reduces the mass flow of air through the engine core, and this, naturally, reduces the engine thrust at a constant TET. The thrust vs Mach number relationship for a constant TET thus changes when bleed air is taken into account. Hence, an investigation was conducted to assess the impact of bleed air extracted from the HP compressor on the available thrust for two-shaft engines. By progressively increasing the bleed air mass flow (defined as a percent of the core mass flow),

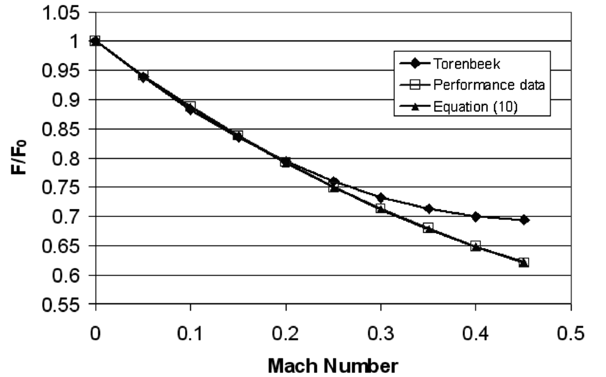

Fig. 7 Difference between Eq. (10) and Torenbeek's approach [1].

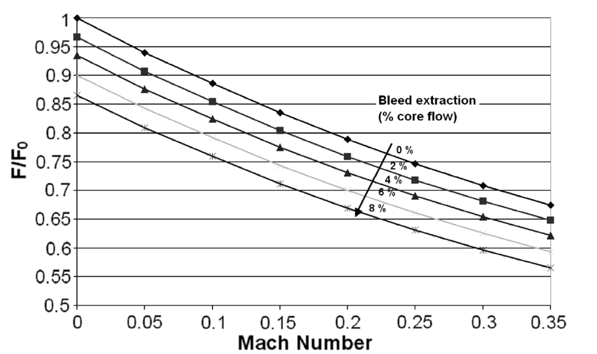

Fig. 8 Reduced takeoff thrust due to bleed air extraction at constant TET.

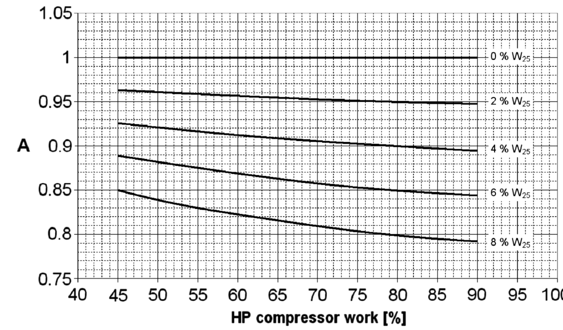

Fig. 9 Influence of bleed air extraction on parameter A.

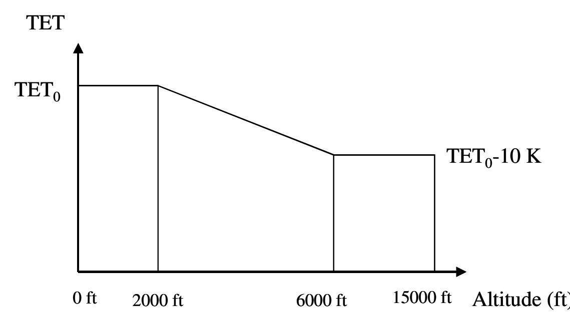

Fig. 10 TET profile as a function of altitude.

the available takeoff thrust was determined for Mach numbers up to 0.35 (Fig. 8).

The main effect that can be seen is a shift of the thrust curve to lower thrust levels compared with the curve without any bleed air taken. It is evident that the change in slope of the curves (compared with the 0% bleed extraction curve) is negligible. This observation leads to a simple method for correcting the thrust vs Mach number relationship expressed by Eq. (10). As proposed earlier in Eq. (8), the parameter A essentially defines the shift of the thrust curve to lower thrust values. Because A depends on the amount of bleed air taken from the compressor and also on the bleed extraction station, an accurate, universally applicable conclusion, valid for all engines, is not possible. However, based on the available engine data, a first approximation of this effect has been summarized in Fig. 9 to give an indication of the impact of bleed air extraction on thrust. This figure has been created with a simple two-shaft engine configuration including the fan and the HP compressor. It should be noted that in the case of additional compressors or a booster in the engine, this figure would no longer be valid.

C. Effect of Altitude on Thrust

The calculations performed so far assume a constant TET vs Mach number relationship. When the variation of thrust vs altitude is considered, this assumption is no longer valid; the TET follows a slope defined by customer requirements and engine restrictions. Generally speaking, the TET decreases with increasing altitude to prevent the NL and nondimensional compressor speeds from rising too much. For the investigation of thrust changes with altitude, a TET assumption has been made according to Fig. 10.

For calculating the thrust values at altitude, Eq. (10) was extended by including empirical factors, designated as A, Z, and X, which are dependant only on the relative air pressure. These factors are calculated from the available engine data. The modified equation is defined as follows:

$$\frac{F}{F_0} = A - \frac{0.377(1+\lambda)}{\sqrt{(1+0.82\lambda)G_0}} ZM + (0.23 + 0.19\sqrt{\lambda})XM^2 \quad (11)$$

where

$$A = -0.4327\left(\frac{p_{\text{amb}}}{p_{\text{amb0}}}\right)^2 + 1.3855\frac{p_{\text{amb}}}{p_{\text{amb0}}} + 0.0472 \quad (12)$$

$$Z = 0.9106\left(\frac{p_{\text{amb}}}{p_{\text{amb0}}}\right)^3 - 1.7736\left(\frac{p_{\text{amb}}}{p_{\text{amb0}}}\right)^2 + 1.8697\frac{p_{\text{amb}}}{p_{\text{amb0}}} \quad (13)$$

$$X = 0.1377\left(\frac{p_{\text{amb}}}{p_{\text{amb0}}}\right)^3 - 0.4374\left(\frac{p_{\text{amb}}}{p_{\text{amb0}}}\right)^2 + 1.3003\frac{p_{\text{amb}}}{p_{\text{amb0}}} \quad (14)$$

As was the case for Eq. (10), Eq. (11) requires G and λ to be known at SLS conditions.

An evaluation was conducted into the accuracy of the derived expression. With values of G and λ known, the accuracy of the thrust calculation was found to be within $\pm 4\%$ compared with the available engine data (Fig. 11).

V. Thrust During Climb

A. Description of Adopted Approach

Predicting an engine's thrust during climb across the flight envelope is a complex problem and requires knowledge of the airplane's climb requirements for the particular application. Because the climb thrust strongly depends on the customer's (i.e., the airplane manufacturer's) requirements, and the engine performance only has a minor influence on the available thrust, simplified assumptions have to be made to describe the engine's thrust behavior. Two assumptions are made:

1) The ambient temperature variation with altitude follows that given by the International Standard Atmosphere (ISA); this is incorporated into the derived equations.

2) The TET varies linearly with altitude, as shown in Fig. 12, providing the maximum available engine thrust without any margin. Corrections are given to estimate thrust values for slow, moderate (i.e., typical), and fast climb applications.

To determine models (empirical equations) that enable the thrust to be estimated for varying Mach number and altitude during a typical climb, the climb path was divided into three segments (herein

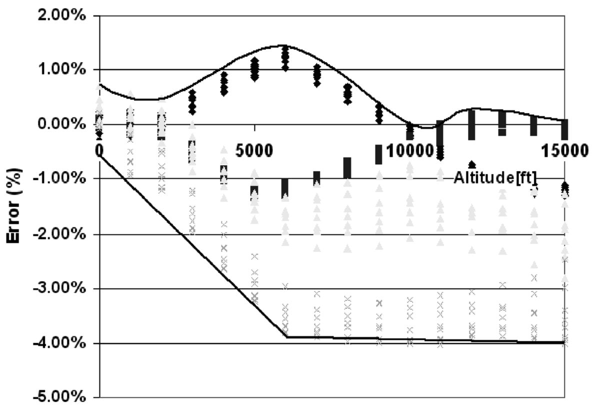

Fig. 11 Deviation of thrust calculated using Eq. (10) and simulated engine performance for four different engines at Mach 0.35, as a function of altitude.

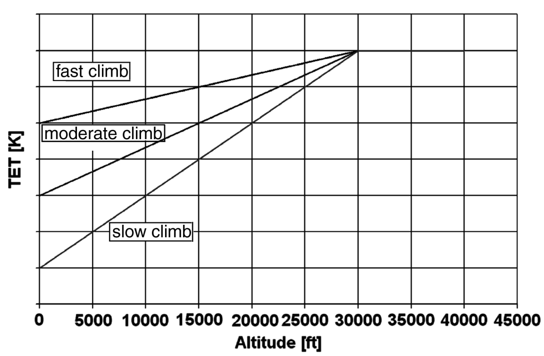

Fig. 12 TET variation with altitude for three climb rates.

called segments 1, 2, and 3), each covering appropriate altitude and speed ranges (Fig. 13). This approach, in which the climb is split into segments, was necessary to achieve an acceptable accuracy of the derived equations in modeling the available date. The range of altitudes and Mach numbers that are covered by the available data is given in Table 1 and Fig. 13, and encompasses the typical climb/speed schedules adopted by airlines in normal operations.

Current approaches for describing the thrust lapse with altitude use the sea level thrust as the reference thrust. For this study, an alternative approach was adopted: the reference thrust was defined at 30,000 ft altitude (ISA conditions) and not at sea level. The reason for this is that climb requirements are generally defined by the airplane manufacturer by specifying the engine performance at the top of climb. This defined thrust can then be translated into an equivalent 30,000 ft thrust [using Eq. (15)], which can be used as the reference thrust for all further climb thrust calculations.

The height at which the climb schedule changes from a constant calibrated airspeed (CAS) climb (segment 2 in Fig. 13) to a constant Mach number climb (segment 3 in Fig. 13) is usually called the "crossover height." This height varies depending on the specific climb/speed schedule, but is generally close to 30,000 ft, and this altitude has been used in the derived thrust model. The aircraft/engine thrust definition also requires a certain reference flight speed (herein called M_{ref}), which is constant above 30,000 ft. The calibrated airspeed that corresponds to M_{ref} at 30,000 ft is designated as V_{casref} and is taken as the climb speed for segment 2. In segment 1,

Table 1 Range of climb paths investigated (climb/speed schedules)

Climb segment	Altitude	Flight speed
1	0–10,000 ft	V_{cas} 200–300 kt
2	10,000–30,000 ft	V_{cas} 250–350 kt
3	30,000–40,000 ft	Mach 0.67–0.91

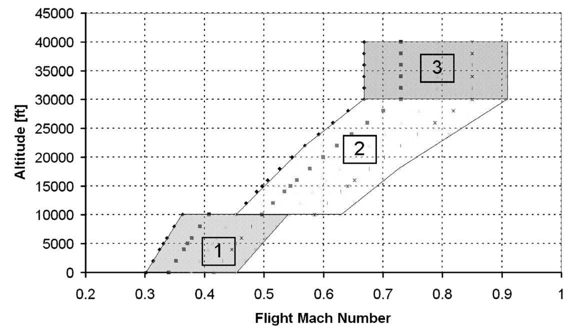

Fig. 13 Climb path (or climb/speed schedule) according to Table 1.

the calibrated airspeed is typically much slower than that adopted for segment 2, and usually corresponds to the air traffic control requirement for a maximum speed (e.g., 250 kt) below 10,000 ft.

The derived thrust relationships are now presented. Working backward from the top of climb, climb segment 3 is considered first.

B. Climb Segment 3

Here, TET is assumed to be constant across altitude and flight speed. The best-fit equation for calculating thrust was evaluated to be

$$\frac{F}{F_{30}} = d\,\ln\left[\frac{p_{\mathrm{amb}}}{p_{\mathrm{amb30}}}\right] + b \tag{15}$$

where the empirical factors b and d take into account the variation of thrust with changing airplane speed. Parameter d can be found in Table 2, and b is defined as

$$b = \left[\frac{M}{M_{\mathrm{ref}}}\right]^{-0.11} \tag{16}$$

C. Climb Segment 2

The second climb segment covers the altitude range from 30,000 down to 10,000 ft, with TET reducing with altitude, according to Fig. 12. The TET is assumed to be constant with changing flight speed.

The equation that best describes the thrust data is

$$\frac{F}{F_{30}} = a\left[\frac{p_{\mathrm{amb}}}{p_{\mathrm{amb30}}}\right]^{-0.355\left(\frac{V_{\mathrm{cas}}}{V_{\mathrm{casref}}}\right)+n} \tag{17}$$

where a is a function of the flight speed, as given by Eq. (18). Note that V_{casref} is the calibrated airspeed corresponding to M_{ref} at 30,000 ft.

$$a = \left[\frac{V_{\mathrm{cas}}}{V_{\mathrm{casref}}}\right]^{-0.1} \tag{18}$$

The exponent n in Eq. (17) takes into account the different TET increments resulting in different climb rates. The values to be used for each climb rate "setting" are given in Table 3.

D. Climb Segment 1

Between 0 and 10,000 ft, the thrust variation with altitude, for a constant calibrated airspeed, is almost linear; hence, the following

Table 2 Parameter d, dependent on flight Mach number

M/M_{ref}	d
0.85	0.73
0.92	0.69
1.00	0.66
1.08	0.63
1.15	0.60

Table 3 Parameter n, dependent on climb rate setting

Climb rate	n
Fast climb	0.97
Moderate climb	0.93
Slow climb	0.89

Table 4 Correction factor m for thrust calculation in climb segment 1

V_{cas}/V_{casref}	m		
	Fast climb	Moderate climb	Slow climb
0.67	0.4	0.39	0.34
0.75	0.39	0.38	0.33
0.83	0.38	0.37	0.32
0.92	0.37	0.36	0.31
1.0	0.36	0.35	0.30

linear function has been used to describe the thrust for this climb segment:

$$\frac{F}{F_{30}} = m\frac{p_{amb}}{p_{amb30}} + \left\{\left[\frac{F_{10}}{F_{30}}\right] - m\left(\frac{p_{amb10}}{p_{amb30}}\right)\right\} \tag{19}$$

Equation (19) requires the thrust at 10,000 ft (i.e., F_{10}) to be first calculated with Eq. (17) for a good starting value. This value of F_{10} has to be calculated for the same flight speed (V_{cas}) that is used to calculate final thrust. The influence of the flight speed and the influence of the climb rate setting are incorporated in the factor m, which is defined in Table 4.

As with the thrust representations derived for takeoff (Sec. IV), the accuracy of the thrust models for climb was similarly investigated by comparing thrust values calculated using Eqs. (15), (17), and (19) with the performance model output for several engines; this resulted in an accuracy of within ±4%, as shown in Fig. 14.

VI. Thrust Specific Fuel Consumption Representation During Cruise

A typical cruise is conducted at constant or nearly constant speed; without acceleration, it follows that the thrust is equal to the airplane's drag. For the purpose of preliminary aircraft design or initial performance analysis, the main engine-related problem concerns the modeling of the fuel consumed during the cruise sector. For that reason, a study was conducted into TSFC models for two-shaft engines.

The typical cruise TSFC s_1 is often specified by the aircraft manufacturer at a reference speed and altitude. However, for several reasons, aircraft are required to leave this particular flight condition. In such cases, the ability to estimate the TSFC variation with changing Mach number and altitude is important. A frequently used model expresses the TSFC as a function of a reference TSFC s_1, the flight Mach number, and the ambient temperature ratio, according to the following power law model (see [8,9], for example):

$$s = s_1\sqrt{\theta}M^n \tag{20}$$

To calculate the TSFC variation with Mach number, s_1 and n have to be known. The determination of the exponent n for modern two-shaft turbofan engines was the main objective of this part of the work. The approach adopted in [9] was to use the maximum cruise thrust condition for estimating n, and hence the thrust was assumed to be constant. In the work herein reported, thrust was considered at a typical cruise condition (the thrust is equal to the airplane's drag at a constant cruise speed) and this changes with Mach number and altitude. A typical thrust variation with flight Mach number, for a

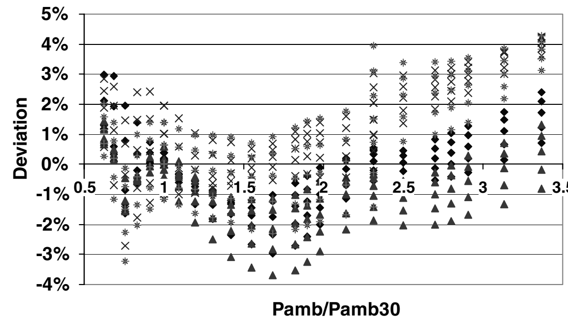

Fig. 14 Deviation of thrust calculated using Eqs. (15–19) and performance data for four different engines, as a function of ambient air pressure ratio.

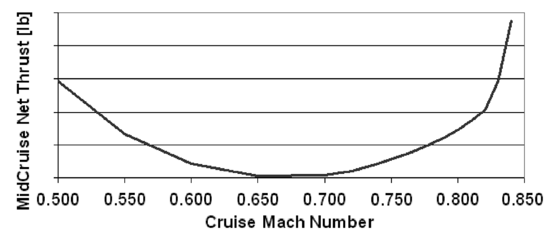

Fig. 15 Typical cruise thrust variation with flight Mach number.

given altitude, is shown in Fig. 15. The investigations performed in this work assume that the reference TSFC s_1 is in the vicinity of the optimum L/D Mach number, which corresponds to the bottom of the loop in Fig. 15.

Because these optimum cruise conditions can be located at different flight Mach numbers and altitudes, Eq. (20) has been generalized in a form given in [10], i.e.,

$$\frac{s}{s^*} = \sqrt{\frac{\theta}{\theta^*}}\left(\frac{M}{M^*}\right)^n \tag{21}$$

where the superscript asterisk (*) defines the reference condition. Herein, the reference condition was selected to be at or close to the optimum L/D speed.

Because of a lack of available aircraft/engine data below 35,000 ft (which is just below the tropopause in the ISA), the influence of the temperature ratio in Eqs. (20) and (21) could not be evaluated. Furthermore, in the stratosphere (where θ is constant), the term $\sqrt{\theta/\theta^*}$ has no influence on the TSFC. Equation (21) was thus modified, and the temperature term was replaced by a factor z, which is defined as

$$z = \left[\frac{F}{F^*}\right]^{-0.1} \quad \text{for } \frac{M}{M^*} > 1 \tag{22}$$

$$z = 1 \quad \text{for } \frac{M}{M^*} \leq 1 \tag{23}$$

where F is any cruise thrust and F^* is the cruise thrust at the reference condition. The thrust at higher flight Mach number has to be known (and is dependent on the aircraft design) to calculate z. Hence, Eq. (21) becomes

$$\frac{s}{s^*} = z\left(\frac{M}{M^*}\right)^n \tag{24}$$

where n has been identified to be a function of F^*/W_1^*, as provided in Fig. 16. Equation (24) is valid for constant level flight at altitudes above 35,000 ft. Because W_1^* (engine inlet mass flow at optimum L/D) strongly depends on the specific engine design, only a rough approximation is possible; a simplified representation of W_1^* vs fan diameter D_{fan} is given in Fig. 17. This correlation between inlet mass flow and fan diameter has been established using information

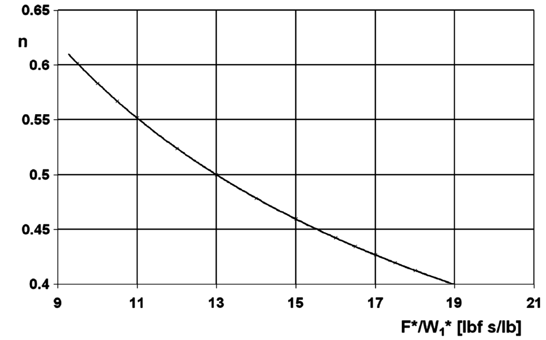

Fig. 16 Exponent *n* dependant on F^*/W_1^*.

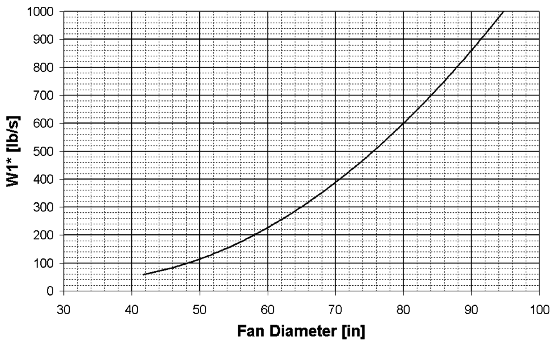

Fig. 17 Simplified dependency of inlet mass flow with fan diameter at cruise.

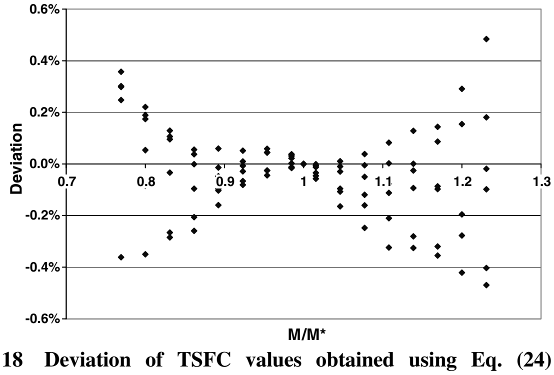

Fig. 18 Deviation of TSFC values obtained using Eq. (24) and performance data including all investigated engines.

collected from publicly available engine data from different engine manufacturers.

The deviation between the TSFC values obtained using Eq. (24) and actual performance data is given in Fig. 18; the results indicate that the deviation is less than 0.5% for the engines investigated. The equations have been validated for a range of Mach number given by $M^* \pm 20\%$ and for altitudes from 35,000 to 47,000 ft. As 35,000 ft is just below the stratosphere in the ISA, and as the temperature variation above 35,000 ft is negligible, the potential influence of the term $\sqrt{\theta}$, as given in Eq. (20), could not be investigated. Note that Eq. (24) is only valid for a constant aircraft weight, and hence for a constant altitude.

VII. Conclusions

1) Two-shaft turbofan performance data have been used to evaluate previously published performance models (empirical equations) that describe aeroengine thrust for maximum takeoff and maximum climb, and TSFC during cruise. Four different performance engine models of different layout, thrust range, and technology standard have been used showing that the equations presented in this publication are generally valid for two-shaft turbofan engines in the thrust range of 10,000 up to 25,000 lbf. The bypass ratio of the investigated engine models was in the range of 3–9, and the overall pressure ratio range was 18–36.

2) To describe the maximum takeoff thrust, an equation has been developed to represent the thrust variation against flight Mach number and altitude. This equation [Eq. (10)] is based on the general approach given by Torenbeek [1], but the validity of the results has been extended to higher Mach numbers. Comparing the thrust values for the investigated engines to the values obtained from Eq. (10) produced a correlation that was better than $\pm 1\%$ for flight speeds up to Mach 0.4.

3) The effect of bleed air extraction and altitude on takeoff thrust has been quantified by introducing an additional factor (herein defined as A) into the thrust vs Mach number model [Eqs. (8–11)].

4) To describe the maximum climb thrust, a new approach has been devised in which the reference thrust is taken as the thrust at 30,000 ft, and, to improve the accuracy of these relatively simple expressions, the climb path has been divided into three segments (corresponding to the typical climb/speed schedule adopted for flight operations) with an equation for each segment. This approach yielded results that varied by less than 4.5% from the available engine performance data.

5) For the cruise segment, the TSFC power law model [Eq. (20)] was found not to be a particularly accurate representation at typical cruise thrust. The resulting investigation produced Eqs. (21–24), which were found to provide a better model of the TFSC for altitudes equal to or greater than 35,000 ft (which is the region in which cruise mostly takes place). The accuracy of these equations was found to be within $\pm 0.5\%$ compared with the available engine performance data.

References

[1] Torenbeek, E., *Synthesis of Subsonic Airplane Design*, Delft Univ. Press, Delft, The Netherlands, 1982.

[2] Anderson, J. D., Jr., *Airplane Performance and Design*, McGraw–Hill, New York, 1999.

[3] Eshelby, M. E., *Airplane Performance: Theory and Practice*, Arnold, London, 2000.

[4] Asselin, M., *Introduction to Aircraft Performance*, AIAA, Reston, VA, 1997.

[5] Ojha, S. K., *Flight Performance of Airplane*, AIAA, Reston, VA, 1995.

[6] Raymer, D., *Airplane Design: A Conceptual Approach*, 4th ed., AIAA, Reston, VA, 2006.

[7] Howe, D., *Aircraft Conceptual Design Synthesis*, Professional Engineering Publishing, London, 2000.

[8] Mair, W. A., and Birdsall, D. L., *Aircraft Performance*, Cambridge Univ. Press, Cambridge, England, U.K., 1992.

[9] "Approximate Methods for Estimation of Cruise Range and Endurance: Aeroplanes With Turbo-Jet and Turbo-Fan Engines," IHS ESDU International, ESDU 73019, 1982.

[10] Martinez-Val, R., and Perez, E., "Optimum Cruise Lift Coefficient in Initial Design of Jet Aircraft," *Journal of Aircraft*, Vol. 29, No. 4, 1992, pp. 712–714.

[11] Braeunling, W., *Flugzeugtriebwerke*, Springer–Verlag, Heidelberg, Germany, 2001.